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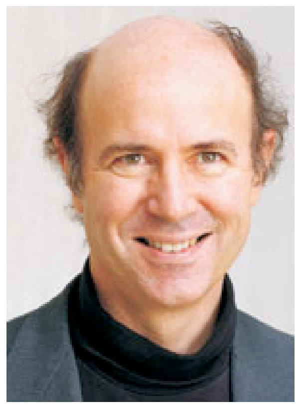
Whence the Force of *F = ma?* I: Culture Shock

Frank Wilczek



When I was a student, the subject that gave me the most trouble was classical mechanics. That always struck me as peculiar, because I had no trouble learning more advanced subjects, which were supposed to be harder. Now I think I've figured it out. It was a case of culture shock. Coming from mathematics, I was expecting an algorithm. Instead I encountered something quite different— a sort of culture, in fact. Let me explain.

Problems with *F = ma*

Newton's second law of motion, $F = ma$, is the soul of classical mechanics. Like other souls, it is insubstantial. The right–hand side is the product of two terms with profound meanings. Acceleration is a purely kinematical concept, defined in terms of space and time. Mass quite directly reflects basic measurable properties of bodies (weights, recoil velocities). The left–hand side, on the other hand, has no independent meaning. Yet clearly Newton's second law is full of meaning, by the highest standard: It proves itself useful in demanding situations. Splendid, unlikely looking bridges, like the Erasmus Bridge (known as the Swan of Rotterdam), do bear their loads; spacecraft do reach Saturn.

The paradox deepens when we consider force from the perspective of modern physics. In fact, the concept of force is conspicuously absent from our most advanced formulations of the basic laws. It doesn't appear in Schrödinger's equation, or in any reasonable formulation of quantum field theory, or in the foundations of general relativity. Astute observers commented on this trend to eliminate force even before the emergence of relativity and quantum mechanics.

In his 1895 *Dynamics*, the prominent physicist Peter G. Tait, who was a close friend and collaborator of Lord Kelvin and James Clerk Maxwell, wrote

"In all methods and systems which involve the idea of force there is a leaven of artificiality. . . . there is no necessity for the introduction of the word "force" nor of the sense–suggested ideas on which it was originally based."[1]

Particularly striking, since it is so characteristic and so over–the–top, is what Bertrand Russell had to say in his 1925 popularization of relativity for serious intellectuals, *The ABC of Relativity*:

"If people were to learn to conceive the world in the new way, without the old notion of "force," it would alter not only their physical imagination, but probably also their morals and politics. . . . In the Newtonian theory of the solar system, the sun seems like a monarch whose behests the planets have to obey. In the Einsteinian world there is more individualism and less government than in the Newtonian."[2]

The 14th chapter of Russell's book is entitled "The Abolition of Force."

If $F = ma$ is formally empty, microscopically obscure, and maybe even morally suspect, what's the source of its undeniable power?

The culture of force

To track that source down, let's consider how the formula gets used.

A popular class of problems specifies a force and asks about the motion, or vice versa. These problems look like physics, but they are exercises in differential equations and geometry, thinly disguised. To make contact with physical reality, we have to make assertions about the forces that actually occur in the world. All kinds of assumptions get snuck in, often tacitly.

The zeroth law of motion, so basic to classical mechanics that Newton did not spell it out explicitly, is that mass is conserved. The mass of a body is supposed to be independent of its velocity and of any forces imposed on it; also total mass is neither created nor destroyed, but only redistributed, when bodies interact. Nowadays, of course, we know that none of that is quite true.

Newton's third law states that for every action there's an equal and opposite reaction. Also, we generally assume that forces do not depend on velocity. Neither of those assumptions is quite true either; for example, they fail for magnetic forces between charged particles.

When most textbooks come to discuss angular momentum, they introduce a fourth law, that forces between bodies are directed along the line that connects them. It is introduced in order to "prove" the conservation of angular momentum. But this fourth law isn't true at all for molecular forces.

Other assumptions get introduced when we bring in forces of constraint, and friction.

I won't belabor the point further. To anyone who reflects on it, it soon becomes clear that $F = ma$ by itself does not provide an algorithm for constructing the mechanics of the world. The equation is more like a common language, in which different useful insights about the mechanics of the world can be expressed. To put it another way, there is a whole culture involved in the interpretation of the symbols. When we learn mechanics, we have to see lots of worked examples to grasp properly what force really means. It is not just a matter of building up skill by practice; rather, we are imbibing a tacit culture of working assumptions. Failure to appreciate this is what got me in trouble.

The historical development of mechanics reflected a similar learning process. Isaac Newton scored his greatest and most complete success in planetary astronomy, when he discovered that a single force of quite a simple form dominates the story. His attempts to describe the mechanics of extended bodies and fluids in the second book of *The Principia* were path breaking but not definitive, and he hardly touched the more practical side of mechanics. Later physicists and mathematicians including notably Jean d'Alembert (constraint and contact forces), Charles Coulomb (friction), and Leonhard Euler (rigid, elastic, and fluid bodies) made fundamental contributions to what we now comprehend in the culture of force.

Physical, psychological origins

Many of the insights embedded in the culture of force, as we've seen, aren't completely correct. Moreover, what we now think are more correct versions of the laws of physics won't fit into its language easily, if at all. The situation begs for two probing questions: How can this culture continue to flourish? Why did it emerge in the first place?

For the behavior of matter, we now have extremely complete and accurate laws that in principle cover the range of phenomena addressed in classical mechanics and, of course, much more. Quantum electrodynamics (QED) and quantum chromodynamics (QCD) provide the basic laws for building up material bodies and the nongravitational forces between them, and general relativity gives us a magnificent account of gravity. Looking down from this exalted vantage point, we can get a clear perspective on the territory and boundaries of the culture of force.

Compared to earlier ideas, the modern theory of matter, which really only emerged during the 20th century, is much more specific and prescriptive. To put it plainly, you have much less freedom in interpreting the symbols. The equations of QED and QCD form a closed logical system: They inform you what bodies can be produced at the same time as they prescribe their behavior; they govern your measuring devices — and you, too!— thereby defining what questions are well posed physically; and they provide answers to such questions — or at least algorithms to arrive at the answers. (I'm well aware that QED + QCD is not a complete theory of nature, and that, in practice, we can't solve the equations very well.) Paradoxically, there is much less interpretation, less culture involved in the foundations of modern physics than in earlier, less complete syntheses. The equations really do speak for themselves: They are algorithmic.

By comparison to modern foundational physics, the culture of force is vaguely defined, limited in scope, and approximate. Nevertheless it survives the competition, and continues to flourish, for one overwhelmingly good reason: It is much easier to work with. We really do not want to be picking our way through a vast Hilbert space, regularizing and renormalizing ultraviolet divergences as we go, then analytically continuing Euclidean Green's functions defined by a limiting procedure, . . . working to discover nuclei that clothe themselves with electrons to make atoms that bind together to make solids, . . . all to describe the collision of two billiard balls. That would be lunacy similar in spirit to, but worse than, trying to do computer graphics from scratch, in machine code, without the benefit of an operating system. The analogy seems apt: Force is a flexible construct in a high–level language, which, by shielding us from irrelevant details, allows us to do elaborate applications relatively painlessly.

Why is it possible to encapsulate the complicated deep structure of matter? The answer is that matter ordinarily relaxes to a stable internal state, with high energetic or entropic barriers to excitation of all but a few degrees of freedom. We can focus our attention on those few effective degrees of freedom; the rest just supply the stage for the actors.

While force itself does not appear in the foundational equations of modern physics, energy and momentum certainly do, and force is very closely related to them: Roughly speaking, it's the space derivative of the former and the time derivative of the latter (and $F = ma$ just states the consistency of those definitions!). So the concept of force is not quite so far removed from modern foundations as Tait and Russell insinuate: It may be gratuitous, but it is not bizarre. Without changing the content of classical mechanics, we can cast it in Lagrangian terms, wherein force no longer appears as a primary concept. But that's really a technicality; the deeper questions remains: What aspects of fundamentals does the *culture* of force reflect? What approximations lead to it?

Some kind of approximate, truncated description of the dynamics of matter is both desirable and feasible because it is easier to use and focuses on the relevant. To explain the rough validity and origin of specific concepts and idealizations that constitute the culture of force, however, we must consider their detailed content. A proper answer, like the culture of force itself, must be both complicated and open–ended. The molecular explanation of friction is still very much a research topic, for example. I'll discuss some of the simpler aspects, addressing the issues raised above, in my next column, before drawing some larger conclusions.

Here I conclude with some remarks on the psychological question, why force was— and usually still is— introduced in the foundations of mechanics, when from a logical point of view energy would serve at least equally well, and arguably better. The fact that changes in momentum— which correspond, by definition, to forces— are visible, whereas changes in energy often are not, is certainly a major factor. Another is that, as active participants in statics— for example, when we hold up a weight— we definitely feel we are doing something, even though no mechanical work is performed. Force is an abstraction of this sensory experience of exertion. D'Alembert's substitute, the virtual work done in response to small displacements, is harder to relate to. (Though ironically it is a sort of virtual work, continually made real, that explains our exertions. When we hold a weight steady, individual muscle fibers contract in response to feedback signals they get from spindles; the spindles sense small displacements, which must get compensated before they grow.[4]) Similar reasons may explain why Newton used force. A big part of the explanation for its continued use is no doubt (intellectual) inertia.

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***Frank Wilczek** is the Herman Feshbach Professor of Physics at the Massachusetts Institute of Technology in Cambridge.*

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Whence the Force of $F = ma$? II: Rationalizations

Frank Wilczek


Wilczek

In my previous column (Physics Today, October 2004, page 11), I discussed how assumptions about *F* and *m* give substance to the spirit of $F = ma$. I called this set of assumptions the culture of force. I mentioned that several elements of the culture, though often presented as "laws," appear rather strange from the perspective of modern physics. Here I discuss how, and under what circumstances, some of those assumptions emerge as consequences of modern fundamentals—or don't!

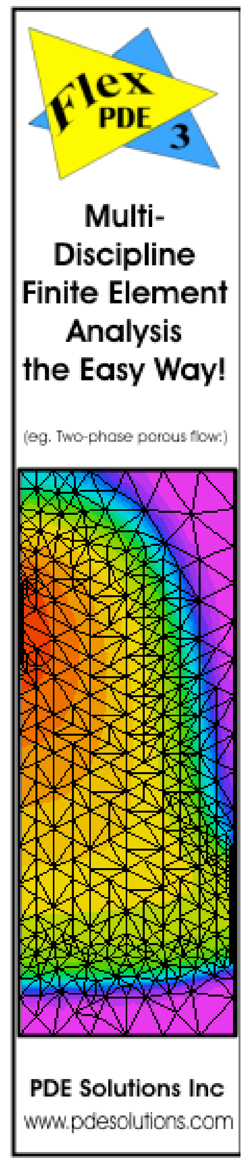
Critique of the zeroth law

Ironically, it is the most primitive element of the culture of force—the zeroth law, conservation of mass—that bears the subtlest relationship to modern fundamentals.

Is the conservation of mass as used in classical mechanics a consequence of the conservation of energy in special relativity? Superficially, the case might appear straightforward. In special relativity we learn that the mass of a body is its energy at rest divided by the speed of light squared ($m = E/c^2$); and for slowly moving bodies, it is approximately that. Since energy is a conserved quantity, this equation appears to supply an adequate candidate, E/c^2, to fill the role of mass in the culture of force.

That reasoning won't withstand scrutiny, however. The gap in its logic becomes evident when we consider how we routinely treat reactions or decays involving elementary particles.

To determine the possible motions, we must explicitly specify the mass of each particle coming in and of each particle going out. Mass is a property of isolated particles, whose masses are intrinsic properties—that is, all protons have one mass, all electrons have another, and so on. (For experts: "Mass" labels irreducible representations of the Poincaré group.) There is no separate principle of mass conservation. Rather, the energies and momenta of such particles are given in terms of their masses and velocities, by well–known formulas, and we constrain the motion by imposing conservation of energy and momentum. In general, it is simply not true that the sum of



the masses of what goes in is the same as the sum of the masses of what goes out.

Of course when everything is slowly moving, then mass does reduce to approximately E/c^2. It might therefore appear as if the problem, that mass as such is not conserved, can be swept under the rug, for only inconspicuous (small and slowly moving) bulges betray it. The trouble is that as we develop mechanics, we want to focus on those bulges. That is, we want to use conservation of energy again, subtracting off the mass–energy exactly (or rather, in practice, ignoring it) and keeping only the kinetic part $E - mc^2 \cong 1/2\, mv^2$. But you can't squeeze two conservation laws (for mass and nonrelativistic energy) out of one (for relativistic energy) honestly. Ascribing conservation of mass to its approximate equality with E/c^2 begs an essential question: Why, in a wide variety of circumstances, is mass–energy accurately walled off, and not convertible into other forms of energy?

To illustrate the problem concretely and numerically, consider the reaction $^2H + {}^3H \rightarrow {}^4He + n$, which is central for attempts to achieve controlled fusion. The total mass of the deuterium plus tritium exceeds that of the alpha plus neutron by 17.6 MeV. Suppose that the deuterium and tritium are initially at rest. Then the alpha emerges at .04 c; the neutron at .17 c.

In the (D,T) reaction, mass is not accurately conserved, and (nonrelativistic) kinetic energy has been produced from scratch, even though no particle is moving at a speed very close to the speed of light. Relativistic energy is conserved, of course, but there is no useful way to divide it up into two pieces that are separately conserved. In thought experiments, by adjusting the masses, we could make this problem appear in situations where the motion is arbitrarily slow. Another way to keep the motion slow is to allow the liberated mass–energy to be shared among many bodies.

Recovering the zeroth law

Thus, by licensing the conversion of mass into energy, special relativity nullifies the zeroth law, in principle. Why is Nature so circumspect about exploiting this freedom? How did Antoine Lavoisier, in the historic experiments that helped launch modern chemistry, manage to reinforce a central principle (conservation of mass) that isn't really true?

Proper justification of the zeroth law requires appeal to specific, profound facts about matter.

To explain why most of the energy of ordinary matter is accurately locked up as mass, we must first appeal to some basic properties of nuclei, where almost all the mass resides. The crucial properties of nuclei are persistence and dynamical isolation. The persistence of individual nuclei is a consequence of baryon number and electric charge conservation, and the properties of nuclear forces, which result in a spectrum of quasi–stable isotopes. The physical separation of nuclei and their mutual electrostatic repulsion—Coulomb barriers—guarantee their approximate dynamical isolation. That approximate dynamical isolation is rendered completely effective by the substantial energy gaps between the ground state of a nucleus and its excited states. Since the internal energy of a nucleus cannot change by a little bit, then in response to small perturbations, it doesn't change at all.

Because the overwhelming bulk of the mass–energy of ordinary matter is concentrated in nuclei, the isolation and integrity of nuclei—their persistence and lack of effective internal structure—go most of the way toward justifying the zeroth law. But note that to get this far, we needed to appeal to quantum theory and special aspects of nuclear phenomenology! For it is quantum theory that makes the concept of energy gaps available, and it is only particular aspects of nuclear forces that insure substantial gaps above the ground state. If it were possible for nuclei to be very much larger and less structured—like blobs of liquid or gas—the gaps would be small, and the mass–energy would not be locked up so completely.

Radioactivity is an exception to nuclear integrity, and more generally the assumption of dynamical isolation goes out the window in extreme conditions, such as we study in nuclear and particle physics. In those circumstances, conservation of mass simply fails. In the common decay $\pi^0 \rightarrow \gamma\gamma$, for example, a massive π^0 particle evolves into photons of zero mass.

The mass of an individual electron is a universal constant, as is its charge.Electrons do not support internal excitations, and the number of electrons is conserved (if we ignore weak interactions and pair creation). These facts are ultimately rooted in quantum field theory. Together, they guarantee the integrity of electron mass–energy.

In assembling ordinary matter from nuclei and electrons, electrostatics plays the dominant role. We learn in quantum theory that the active, outer–shell electrons move with velocities of order $\alpha c = e^2/4\pi\hbar \approx .007\, c$. This indicates that the energies in play in chemistry are of order $m_e(\alpha c)^2/m_e c^2 = \alpha^2 \approx 5 \times 10^{-5}$ times the electron mass–energy, which in turn is a small fraction of the nuclear mass–energy. So chemical reactions change the mass–energy only at the level of parts per billion, and Lavoisier rules!

Note that inner–shell electrons of heavy elements, with velocities of order $Z\alpha$, can be relativistic. But the inner core of a heavy atom—nucleus plus inner electron shells—ordinarily retains its integrity, because it is spatially isolated and has a large energy gap. So the mass–energy of the core is conserved, though it is *not* accurately equal to the sum of the mass–energy of its component electrons and nucleus.

Putting it all together, we justify Isaac Newton's zeroth law for ordinary matter by means of the integrity of nuclei, electrons, and heavy atom cores, together with the slowness of the motion of these building blocks. The principles of quantum theory, leading to large energy gaps, underlie the integrity; the smallness of α, the fine–structure constant, underlies the slow motion.

Newton defined mass as "quantity of matter," and assumed it to be conserved. The connotation of his phrase, which underlies his assumption, is that the building blocks of matter are rearranged, but neither created nor destroyed, in physical processes; and that the mass of a body is the sum of the masses of its building blocks. We've now seen, from the perspective of modern foundations, why ordinarily these assumptions form an excellent approximation, if we take the building blocks to be nuclei, heavy atom cores, and electrons.

It would be wrong to leave the story there, however. For with our next steps in analyzing matter, we depart from this familiar ground: first off a cliff, then into glorious flight. If we try to use more basic building blocks (protons and neutrons) instead of nuclei, then we discover that the masses don't add accurately. If we go further, to the level of quarks and gluons, we can largely derive the mass of nuclei from pure energy, as I've discussed in earlier columns.

Mass and gravity

On the face of it, this complex and approximate justification of the mass concept used in classical mechanics poses a paradox: How does this rickety construct manage to support stunningly precise and successful predictions in celestial mechanics? The answer is that it is bypassed. The forces of celestial mechanics are gravitational, and so proportional to mass, and m cancels from the two sides of $F = ma$. This cancellation in the equation for motion in response to gravity becomes a foundational principle in general relativity, where the path is identified as a geodesic in curved spacetime, with no mention of mass.

In contrast to a particle's *response* to gravity, the gravitational *influence* that the particle exerts is only approximately proportional to its mass; the rigorous version of Einstein's field equation relates spacetime curvature to energy–momentum density. As far as gravity is concerned, there is no separate measure of quantity of matter apart from energy; that the energy of ordinary matter is dominated by mass–energy is immaterial.

The third and fourth laws

The third and fourth laws are approximate versions of conservation of momentum and conservation of angular momentum, respectively. (Recall that the fourth law stated that all forces are two–body central forces.) In the modern foundations of physics these great conservation laws reflect the symmetry of physical laws under translation and rotation symmetry. Since these conservation laws are more accurate and profound than the assumptions about forces commonly used to "derive" them, those assumptions have truly become anachronisms. I believe that they should, with due honors, be retired.

Newton argued for his third law by observing that a system with unbalanced internal forces would begin to accelerate spontaneously, "which is never observed." But this argument really motivates the conservation of momentum directly. Similarly, one can "derive" conservation of angular momentum from the observation that bodies don't spin up spontaneously. Of course, as a matter of pedagogy, one would point out that action–reaction systems and two–body central forces provide especially simple ways to satisfy the conservation laws.

Tacit simplicities

Some tacit assumptions about the simplicity of F are so deeply embedded that we easily take them for granted. But they have profound roots.

In calculating the force, we take into account only nearby bodies. Why can we get away with that? Locality in quantum field theory, which deeply embodies basic requirements of special relativity and quantum mechanics, gives us expressions for energy and momentum at a point—and thereby for force—that depend only on the position of bodies near that point. Even so–called long–range electric and gravitational forces (actually $1/r^2$—still falling rapidly with distance) reflect the special properties of locally coupled gauge fields and their associated covariant derivatives.

Similarly, the absence of significant multibody forces is connected to the fact that sensible (renormalizable) quantum field theories can't support them.

In this column I've stressed, and maybe strained, the relationship between the culture of force and modern fundamentals. In the final column of this series, I'll discuss its importance both as a continuing, expanding endeavor and as a philosophical model.

Frank Wilczek *is the Herman Feshbach Professor of Physics at the Massachusetts Institute of Technology in Cambridge.*

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Whence the Force of $F = ma$? III: Cultural Diversity

Frank Wilczek

The concept of force, as we have seen, defines a culture. In the previous columns of this series (PHYSICS TODAY, October 2004, page 11, and December 2004, page 10) I've indicated how $F = ma$ acquires meaning through interpretation of—that is, additional assumptions about—F. This body of interpretation is a sort of folklore. It contains both approximations that we can derive, under appropriate conditions, from modern foundations, and also rough generalizations (such as "laws" of friction and of elastic behavior) abstracted from experience.

In the course of that discussion it became clear that there is also a smaller, but nontrivial, culture around m. Indeed, the conservation of m for ordinary matter provides an excellent, instructive example of an emergent law. It captures in a simple statement an important consequence of broad regularities whose basis in modern fundamentals is robust but complicated. In modern physics, the idea that mass is conserved is drastically false. A great triumph of modern quantum chromodynamics (QCD) is to build protons and neutrons, which contribute more than 99% of the mass of ordinary matter, from gluons that have exactly zero mass, and from u and d quarks that have very small masses. To explain from a modern perspective why conservation of mass is often a valid approximation, we need to invoke specific, deep properties of QCD and quantum electrodynamics (QED), including the dynamical emergence of large energy gaps in QCD and the smallness of the fine structure constant in QED.

Isaac Newton and Antoine Lavoisier knew nothing of all this, of course. They took conservation of mass as a fundamental principle. And they were right to do so, because by adopting that principle they were able to make brilliant progress in the analysis of motion and of chemical change. Despite its radical falsity, their principle was, and still is, an adequate basis for many quantitative applications. To discard it is unthinkable. It is an invaluable cultural artifact and a basic insight into the way the world works despite—indeed, in part, because of—its emergent character.

Frank Wilczek *is the Herman Feshbach Professor of Physics at the Massachusetts Institute of Technology in Cambridge.*

The culture of *a*

What about a? There's a culture attached to acceleration, as well. To obtain a, we are instructed to consider the change of the position of a body in space as a function of time, and to take the second derivative. This prescription, from a modern perspective, has severe problems.

In quantum mechanics, bodies don't have definite positions. In quantum field theory, they pop in and out of existence. In quantum gravity, space is fluctuating and time is hard to define. So evidently serious assumptions and approximations are involved even in making sense of a's definition.

Nevertheless, we know very well where we're going to end up. We're going to have an emergent, approximate concept of what a body is. Physical space is going to be modeled mathematically as the Euclidean three-dimensional space $\mathbf{R}^3$ that supports Euclidean geometry. This tremendously successful model of space has been in continuous use for millennia, with applications in surveying and civil engineering that even predate Euclid's formalization.

Time is going to be modeled as the one-dimensional continuum $\mathbf{R}^1$ of real numbers. This model of time, at a topological level, goes into our primitive intuitions that divide the world into past and future. I believe that the metric structure of time—that is, the idea that time can be not only ordered but divided into intervals with definite numerical magnitude—is a much more recent innovation. That idea emerged clearly only with Galileo's use of pendulum clocks (and his pulse!).

The mathematical structures involved are so familiar and fully developed that they can be, and are, used routinely in computer programs. This is not to say they are trivial. They most definitely aren't. The classical Greeks agonized over the concept of a continuum. Zeno's famous paradoxes reflect these struggles. Indeed, Greek mathematics never won through to comfortable algebraic treatment of real numbers. Continuum quantities were always represented as geometric intervals, even though that representation involved rather awkward constructions to implement simple algebraic operations.

The founders of modern analysis (René Descartes, Newton, Gottfried Wilhelm Leibniz, Leonhard Euler, and others) were on the whole much more freewheeling, trusting their intuition while manipulating infinitesimals that lacked any rigorous definition. (In his *Principia*, Newton did operate geometrically, in the style of the Greeks. That is what makes the *Principia* so difficult for us to read today. The *Principia* also contains a sophisticated discussion of derivatives as limits. From that discussion I infer that Newton and possibly other early analysts had a pretty good idea about what it would take to make at least the simpler parts of their work rigorous, but they didn't want to slow down to do it.) Reasonable rigor, at the level commonly taught in mathematics courses today—the much-bemoaned epsilons and deltas—entered into the subject in the 19th century.

"Unreasonable" rigor entered in the early 20th century, when the fundamental notions from which real numbers and geometry are constructed were traced to the level of set theory and ultimately symbolic logic. In their *Principia Mathematica* Bertrand Russell and Alfred Whitehead develop 375 pages of dense mathematics before proving $1 + 1 = 2$. To be fair, their treatment could be slimmed down considerably if attaining that particular result were the ultimate goal. But in any case, an adequate definition of real numbers from symbolic logic involves some hard, complicated work. Having the integers in hand, you then have to define rational numbers and their ordering. Then you must complete them by filling in the holes so that any bounded increasing sequence has a limit. Then finally—this is the hardest part—you must demonstrate that the resulting system supports algebra and is consistent.

Perhaps all that complexity is a hint that the real-number model of space and time is an emergent concept that some day will be derived from physically motivated primitives that are logically simpler. Also, scrutiny of the construction of real numbers suggests natural variants, notably John Conway's surreal numbers, which include infinitesimals (smaller than any rational number!) as legitimate quantities.[1] Might such quantities, whose formal properties seem no less natural and elegant than those of ordinary real numbers, help us to describe nature? Time will tell.

Even the unreasonable rigor of symbolic logic does not reach ideal strictness. Kurt Gödel demonstrated that this ideal is unattainable: No reasonably complex, consistent axiomatic system can be used to demonstrate its own consistency.

But all the esoteric shortcomings in defining and justifying the culture of a clearly arise on an entirely different level from the comparatively mundane, immediate difficulties we have in doing justice to the culture of F. We can translate the culture of a, without serious loss, into C or FORTRAN. That completeness and precision give us an inspiring benchmark.

The computational imperative

Before they tried to do it, most computer scientists anticipated that to teach a computer to play chess like a grand master would be much more challenging than to teach one to do mundane tasks like drive a car safely. Notoriously, experience has proved otherwise. A big reason for that surprise is that chess is algorithmic, whereas driving a car is not. In chess the rules are completely explicit; we know very concretely and unambiguously what the degrees of freedom are and how they behave. Car driving is quite different: Essential concepts like "other driver's expectations" and "pedestrian," when you start to analyze them, quickly burgeon into cultures. I wouldn't trust a computer driver in Boston's streets because it wouldn't know how to interpret the mixture of intimidation and deference that human drivers convey by gestures, maneuvers, and eye contact.

The problem with teaching a computer classical mechanics is, of course, of more than academic interest: We'd like robots to get around and manipulate things; computer gamesters want realistic graphics; engineers and astronomers would welcome smart silicon collaborators—up to a point, I suppose.

The great logician and philosopher Rudolf Carnap made brave, pioneering attempts to make axiomatic systems for elementary mechanics, among many other things.[2] Patrick Hayes issued an influential paper, "Naive Physics Manifesto," challenging artificial-intelligence researchers to codify intuitions about materials and forces in an explicit way.[3] Physics-based computer graphics is a lively, rapidly advancing endeavor, as are several varieties of computer-assisted design. My MIT colleagues Gerald Sussman and Jack Wisdom have developed an intensely computational approach to mechanics,[4] supported every step of the way with explicit programs. The time may be ripe for a powerful synthesis, incorporating empirical properties of specific materials, successful known designs of useful mechanisms, and general laws of mechanical behavior into a fully realized computational culture of $F = ma$. Functioning robots might not need to know a lot of mechanics explicitly, any more than most human soccer players do; but *designing* a functioning robotic soccer player may be a job that can best be accomplished by a very smart and knowledgeable man-machine team.

Blur and focus

An overarching theme of this series has been that the law $F = ma$, which is sometimes presented as the epitome of an algorithm describing nature, is actually not an algorithm that can be applied mechanically (pun intended). It is more like a language in which we can easily express important facts about the world. That's not to imply it is without content. The content is supplied, first of all, by some powerful general statements in that language—such as the zeroth law, the momentum conservation laws, the gravitational force law, the necessary association of forces with nearby sources—and then by the way in which phenomenological observations, including many (though not all) of the laws of material science can be expressed in it easily.

Another theme has been that $F = ma$ is not in any sense an ultimate truth. We can understand, from modern foundational physics, how it arises as an approximation under wide but limited circumstances. Again, that does not prevent it from being extraordinarily useful; indeed, one of its primary virtues is to shield us from the unnecessary complexity of irrelevant accuracy!

Viewed this way, the law of physics $F = ma$ comes to appear a little softer than is commonly considered. It really does bear a family resemblance to other kinds of laws, like the laws of jurisprudence or of morality, wherein the meaning of the terms takes shape through their use. In those domains, claims of ultimate truth are wisely viewed with great suspicion; yet nonetheless we should actively aspire to the highest achievable level of coherence and explicitness. Our physics culture of force, properly understood, has this profoundly modest but practically ambitious character. And once it is no longer statuized, put on a pedestal, and seen in splendid isolation, it comes to appear as an inspiring model for intellectual endeavor more generally.

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