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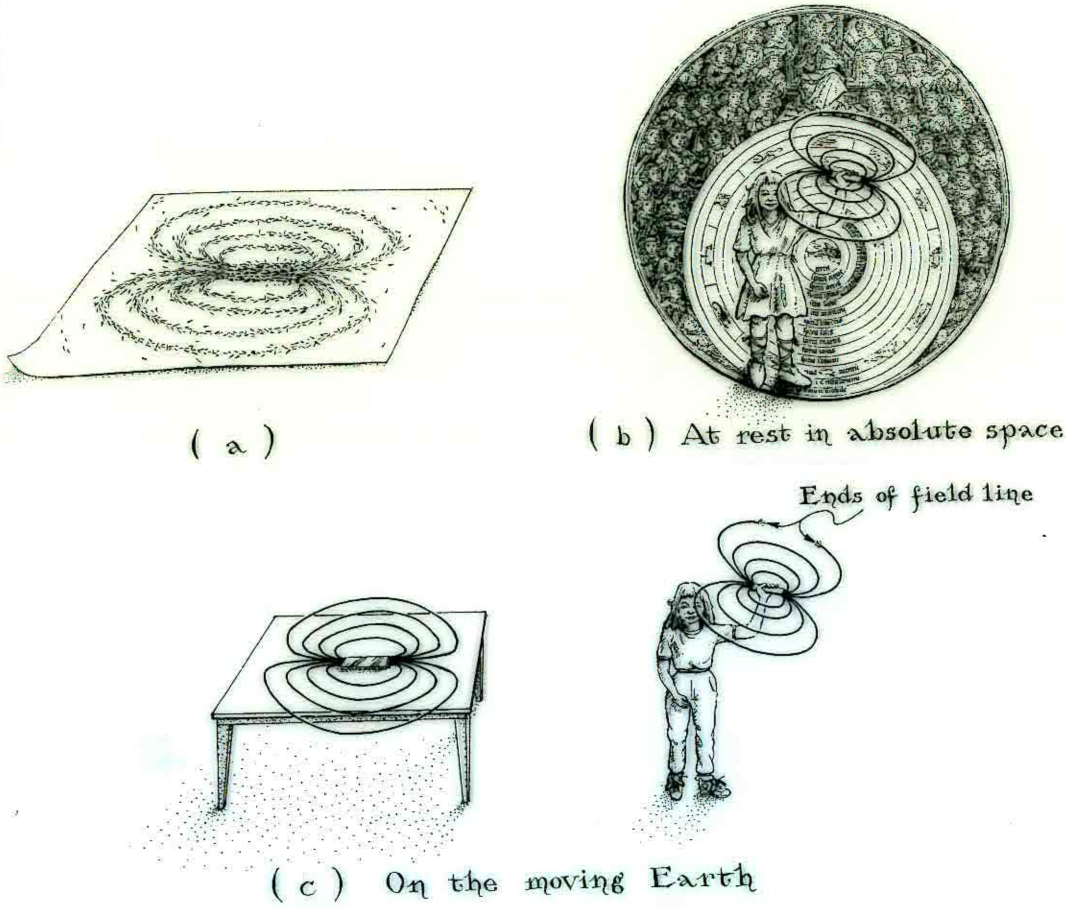


1.1 One of Maxwell's electromagnetic laws, as understood within the framework of nineteenth-century, Newtonian physics: (a) The concept of a magnetic field line: When one places a bar magnet under a sheet of paper and scatters iron filings on top of the sheet, the filings mark out the magnet's field lines. Each field line leaves the magnet's north pole, swings around the magnet and reenters it at the south pole, and then travels through the magnet to the north pole, where it attaches onto itself. The field line is therefore a closed curve, somewhat like a rubber band, without any ends. The statement that "magnetic field lines never have ends" is Maxwell's law in its simplest, most beautiful form. (b) According to Newtonian physics, this version of Maxwell's law is correct no matter what one does with the magnet (for example, even if one shakes it wildly) *so long as one is at rest in absolute space.* No magnetic field line *ever* has any ends, from the viewpoint of someone at rest. (c) When studied by someone riding on the surface of the Earth as it moves through absolute space, Maxwell's law is much more complicated, according to Newtonian physics. If the moving person's magnet sits quietly on a table, then a few of its field lines (about one in a hundred million) will have ends. If the person shakes the magnet wildly, additional field lines (one in a trillion) will get cut temporarily by the shaking, and then will heal, then get cut, then reheal. Although one field line in a hundred million or a trillion with ends was far too few to be discerned in any nineteenth-century physics experiment, the fact that Maxwell's laws predicted such a thing seemed rather complicated and ugly to Lorentz, Poincaré, and Larmor.

ain on the next track has begun to creep out of the sta- on. However, after several moments you realize that it your own train that is moving and that the other train st otionless on its track.

nsider another example. Suppose you are floating a spaceship in interstellar space and another spaceship mes coasting by (Figure 5-17a). You might conclude at it is moving and you are not, but someone in the her ship might be equally sure that you are moving and is not. The principle of relativity says that there is no periment you can perform to decide which ship is mov- g and which is not. This means that there is no such ing as absolute rest—all motion is relative.

Because neither you nor the people in the other aceship could perform any experiment to detect your solute motion through space, the laws of physics must ve the same form in both spaceships. Otherwise, exper- ents would give different results in the two ships, and u could decide who was moving. Thus, a more general y of stating the first postulate refers to these laws of ysics:

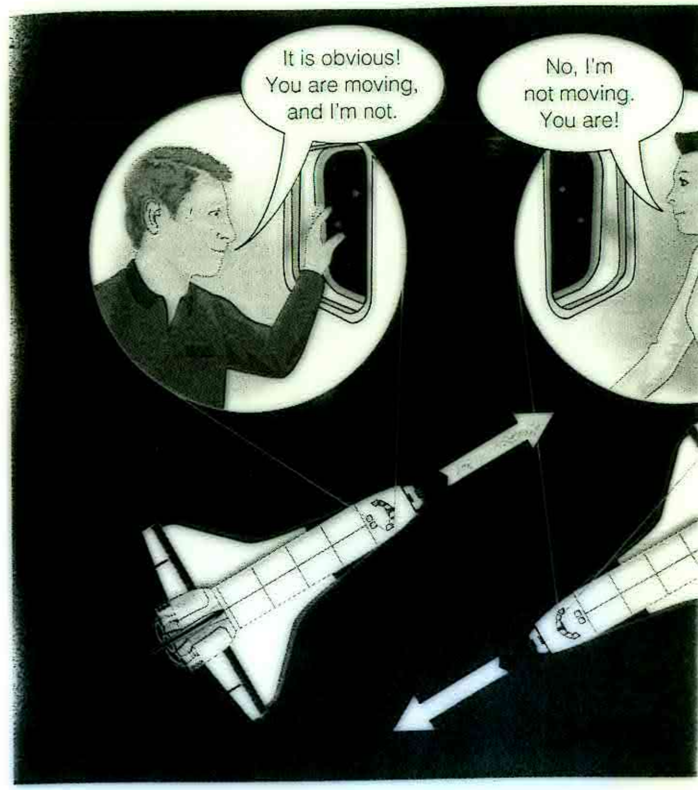
> **First postulate** (alternate version)
> The laws of physics are the same for
> all observers, no matter what their motion,
> so long as they are not *accelerated.*

e words *uniform* and *accelerated* are important. If either aceship were to fire its rockets, then its velocity would ange. The crew of that ship would know it because uld feel the acceleration pressing them into their uches. Accelerated motion, therefore, is different—we n always tell which ship is accelerating and which is t. The postulates of relativity discussed here apply only observers in uniform motion. That is why the theory called **special relativity.**

The first postulate fit with Einstein's conclusion that speed of light must be constant for all observers. No tter how you move, your measurement of the speed of ht has to give the same result (Figure 5-17b). This came the second postulate of special relativity:

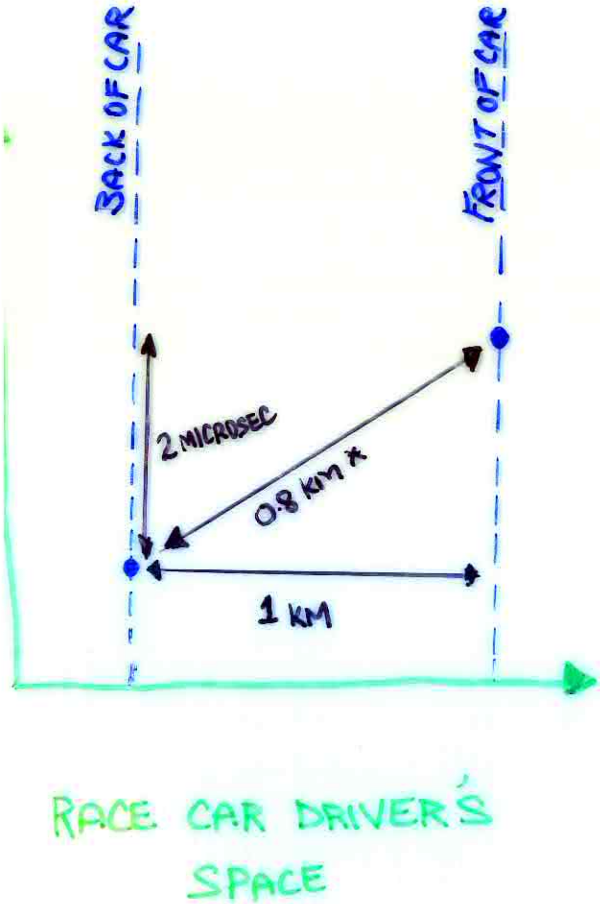
> **Second postulate** The velocity of light
> is constant and will be the same for all
> observers independent of their motion
> relative to the light source.

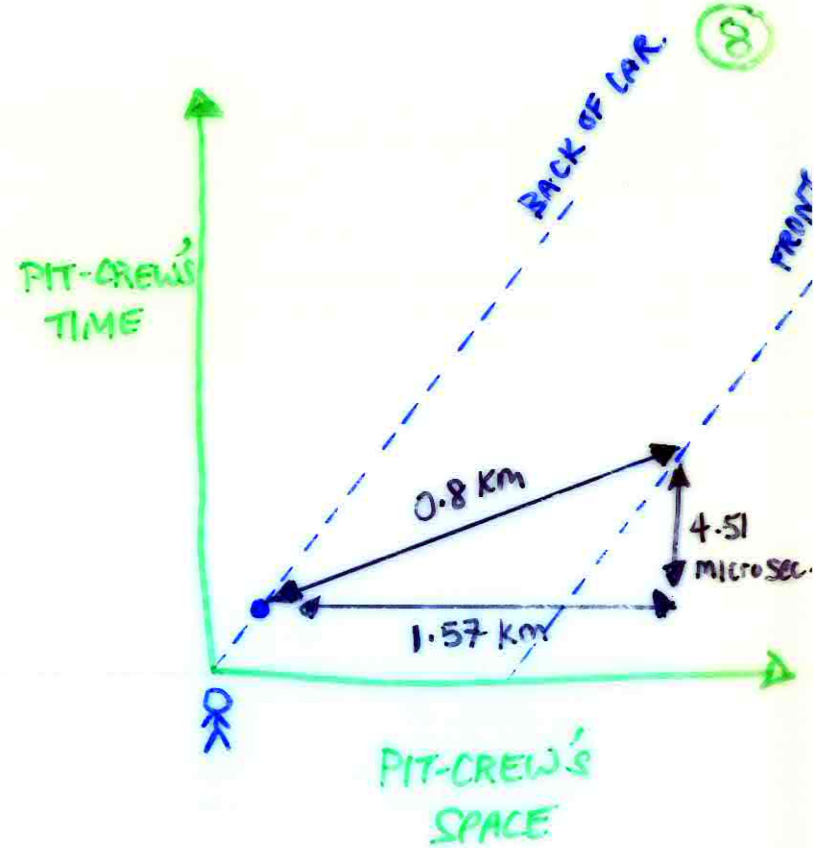
Once Einstein had accepted the basic postulates of ativity, he was led to some startling discoveries. New- n's laws of motion and gravity worked well as long as tances were small and velocities were low. But when begin to think of very large distances or very high locities, Newton's laws are no longer adequate to scribe what happens. Instead, we must use relativistic ysics. For example, special relativity shows that the served mass of a moving particle depends on its veloc- e higher the velocity, the greater the mass of the rticle. This is not significant at low velocities, but it comes very important as the velocity approaches the locity of light. Such increases in mass are observed



a

b





$$S = \sqrt{\Delta l^2 - c^2 \Delta t^2} = \sqrt{(1.0)^2 - (0.6)^2} = 0.8 \text{ km}.$$

FLASHES SEPARATED BY:

DISTANCE: 1 KM (DRIVER) 1.57 KM (PITCREW)

TIME : 2 μs (DRIVER) 4.5 μs (PITCREW)

SPACE-TIME IS 4-DIMENSIONAL

• COUPLE OF OTHER PROBLEMS IN PHYSICS

EINSTEIN'S SOLUTION: SPACE & TIME ARE NOT ABSOLUTE (RELATIVE TO EACH PERSON)

• EACH PERSON ~~SEES~~ MEASURES SPACE DIFFERENTLY & TIME FLOW IS DIFFERENT TOO. IF HE/SHE IS MOVING DIFFERENTLY THAN OTHERS.

TWO BASIC PRINCIPLES:

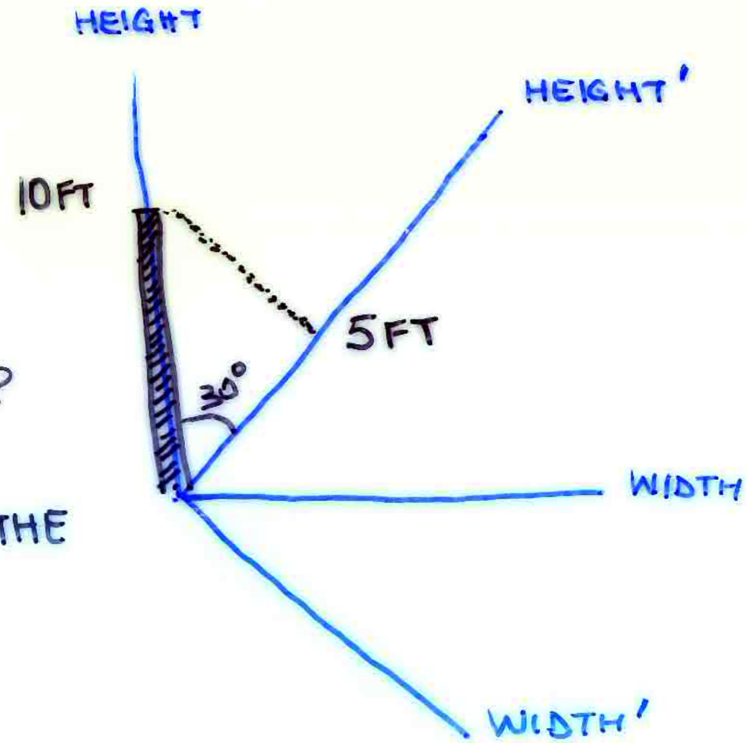
1) SPEED OF LIGHT IS ALWAYS SAME NO MATTER WHO MEASURES IT $c = 3 \times 10^8$ m/s

2) PRINCIPLE OF RELATIVITY: IF YOU ARE MOVING AT CONSTANT VELOCITY, LAWS OF PHYSICS ALWAYS THE SAME NO MATTER HOW FAST YOU GO.

→ SPACE-TIME IS ONE "THING", NOT TWO SEPARATE CONCEPTS.

→ SPACE, TIME IN FACT ARE TWO DIRECTIONS

ANALOGY: LENGTH & WIDTH



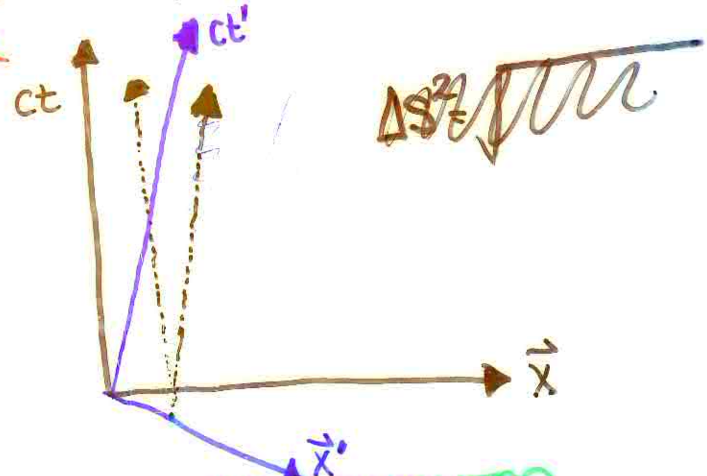
...W HIGH OFF THE GROUND?

...OW LONG IS THE ROD?

→ HEIGHT IN ONE FRAME (FOR ONE PERSON) IS MIXTURE OF HEIGHT AND WIDTH/TILT IN ANOTHER FRAME

→ EXACTLY THE SAME IS TRUE FOR SPACE & TIME

- SPACE OF ONE OBSERVER IS MIXTURE OF SPACE & TIME OF ANOTHER OBSERVER
- SAME FOR TIME

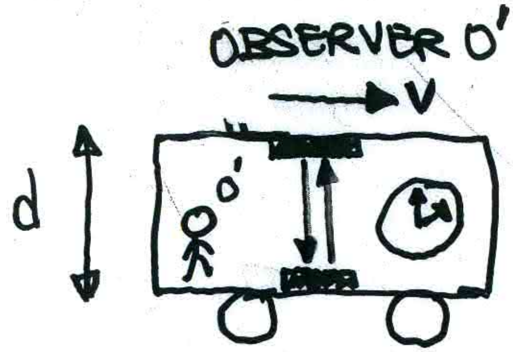


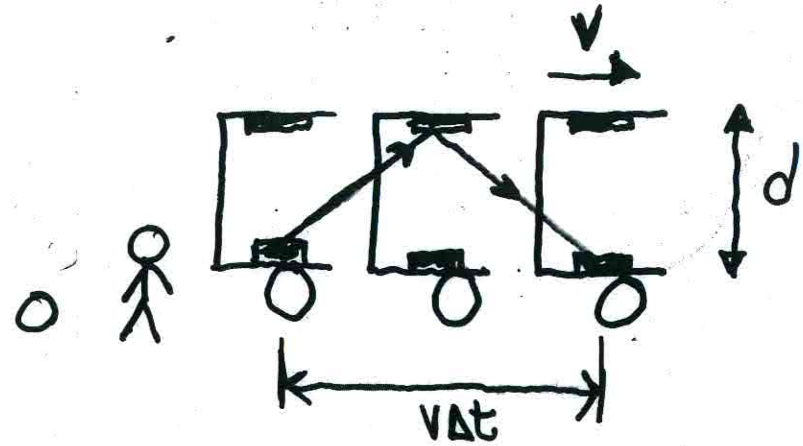
...BUT TOTAL DISTANCE IS SAME FOR ALL OBSERVERS.

$D^2 = H^2 + W^2$ $\qquad$ $D^2 = H'^2 + W'^2$

... ALSO TRUE FOR SPACE-TIME. SPACE-TIME DISTANCE ALWAYS SAM...

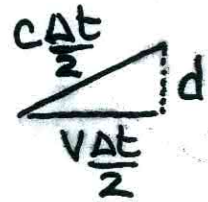
TIME DILATION:





MEASURING TIME INTERVAL BETWEEN TWO EVENTS.

$$\Delta t_{O'} = \frac{2d}{c}$$ INTERVAL IN CLOCK WHERE OBS AT REST W.R.T. CLOCK



$$\Delta t_O = \frac{2d}{c} \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\left(\frac{c\Delta t}{2}\right)^2 = d^2 + \left(\frac{v\Delta t}{2}\right)^2$$

$$= \Delta t_{O'} \gamma$$

INTERVAL IN SAME CLOCK BY MOVING OBS.

$$\Delta t = \frac{2d}{\sqrt{c^2-v^2}} = \frac{2d}{c\sqrt{1-\frac{v^2}{c^2}}}$$

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$$

BECAUSE $\gamma \geq 1$, TIME INTERVAL MEASURED BY OBS <u>MOVING</u> WRT TO CLOCK IS LONGER THAN INTERVAL BETWEEN SAME EVENTS MEASURED BY OBS @ REST W.R.T. CLOCK.

LENGTH CONTRACTION

- ASSUME THAT THERE IS A RULER ON TRAIN AND O' MEASURES ITS LENGTH AS $L_{PROPER} = L_{O'}$

THE MOVING OBS (W.R.T. RULER) - I.E. O - WILL MEASURE ITS LENGTH TO BE $L_p/\gamma = L_O$

AND SINCE $\gamma > 1$ $\quad L_O < L_{O'}$

CONSIDER SPACESHIP TRAVELLING AT SPEED V FROM ONE STAR TO ANOTHER.

TWO OBSERVER : ONE ON EARTH, OTHER IN SHIP
↓
AT REST W.R.T. STARS

EARTH OBS : MEASURES DISTANCE BETWEEN STARS TO BE L_p (EARTH EARTH)

TIME FOR VOYAGE : $\Delta t = L_p/v$

SPACEMAN CLAIMS TO BE AT REST & SEES DESTINATION STAR APPROACHING SHIP AT SPEED V

MEASURES DURATION OF VOYAGE : Δt_p [like $\Delta t_{o'}$]

RELATIVE SPACESHIP CLOCK, EARTH OBS IS MOVING AND MEASURES DURATION $\gamma \Delta t_p$

$\Delta t = \gamma \Delta t_p \qquad \Delta t_p = \Delta t/\gamma$

(Δt_p measured by spaceman; Δt measured on earth.)

SPACEMAN MEASURE VOYAGE DISTANCE TO BE:

$$L = v\Delta t_p = v\frac{\Delta t}{\gamma} = \frac{L_p}{\gamma}$$

SINCE $\gamma > 1$ $\quad L < L_p$

* OBS MOVING AT SPEED V RELATIVE TO OBJ WILL MEASURE ITS LENGTH TO BE SHORTER THAN PROPER LENGTH BY FACTOR $\sqrt{1-v^2/c^2}$

* CONTRACTION TAKES PLACE ONLY ALONG DIRECTION OF MOTION.

the length of the rod as measured in the frame S' is $L' = x_2' - x_1'$. What is the length of the rod measured from S? Because the rod is moving relative to S, care must be taken to measure the x-coordinates x_1 and x_2 of the ends of the rod *at the same time*. Then Eq. (4.16), with $t_1 = t_2$, shows that the length $L = x_2 - x_1$ measured in S may be found from

$$x_2' - x_1' = \frac{(x_2 - x_1) - u(t_2 - t_1)}{\sqrt{1 - u^2/c^2}}$$

or

$$L' = \frac{L}{\sqrt{1 - u^2/c^2}}. \tag{4.28}$$

Because the rod is *at rest* relative to S', L' will be called L_{rest}. Similarly, because the rod is *moving* relative to S, L will be called L_{moving}. Thus Eq. (4.28) becomes

$$L_{\text{moving}} = L_{\text{rest}}\sqrt{1 - u^2/c^2}. \tag{4.29}$$

This equation shows the effect of **length contraction** on a moving rod. It says that length or distance is measured differently by two observers in relative motion. If a rod is moving relative to an observer, that observer will measure a shorter rod than will an observer at rest relative to it. The *longest length*, called the rod's **proper length**, is measured in the rod's rest frame. Only lengths or distances *parallel* to the direction of the relative motion are affected by length contraction; distances perpendicular to the direction of the relative motion are unchanged (c.f. Eqs. 4.17–4.18).

Example 4.1 Cosmic rays from space collide with the nuclei of atoms in Earth's upper atmosphere, producing elementary particles called *muons*. Muons are unstable and decay after an average lifetime $\tau = 2.20 \times 10^{-6}$ s, as measured in a laboratory where the muons are at rest. That is, the number of muons in a given sample should decrease with time according to $N(t) = N_0 e^{-t/\tau}$, where N_0 is the number of muons originally in the sample at time $t = 0$. At the top of Mt. Washington in New Hampshire, a detector counted 563 muons hr^{-1} moving downward at a speed $u = 0.9952c$. At sea level, 1907 m below the first detector, another detector counted 408 muons hr^{-1}.[6]

The muons take $(1.907 \times 10^5 \text{ cm})/(0.9952c) = 6.39 \times 10^{-6}$ s to travel from the top of Mt. Washington to sea level. Thus it might be expected that the number of muons detected per hour at sea level would have been

$$N = N_0 e^{-t/\tau} = 563\, e^{-(6.39\times10^{-6}\text{ s})/(2.20\times10^{-6}\text{ s})} = 31 \text{ muons hr}^{-1}.$$

[6] Details of this experiment can be found in Frisch and Smith (1963).

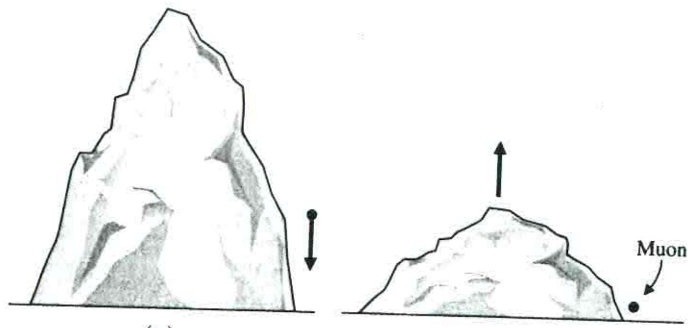


Figure 4.8 Muons moving downward past Mt. Washington. (a) Mountain frame. (b) Muon frame.

This is much less than the 408 muons hr^{-1} actually measured at sea level! How did the muons live long enough to reach the lower detector? The problem with the preceding calculation is that the lifetime of 2.20×10^{-6} s is measured in the muon's rest frame as Δt_{rest}, but the experimenter's clocks on Mt. Washington and below are moving relative to the muons. They measure the muon's lifetime to be

$$\Delta t_{\text{moving}} = \frac{\Delta t_{\text{rest}}}{\sqrt{1 - u^2/c^2}} = \frac{2.20 \times 10^{-6}\ \text{s}}{\sqrt{1 - (0.9952)^2}} = 2.25 \times 10^{-5}\ \text{s},$$

more than *ten* times a muon's lifetime when measured in its own rest frame. The moving muons' clocks run slower, so more of them survive long enough to reach sea level. Repeating the preceding calculation using the muon lifetime as measured by the experimenters gives

$$N = N_0\, e^{-t/\tau} = 563\, e^{-(6.39 \times 10^{-6}\ \text{s})/(2.25 \times 10^{-5}\ \text{s})} = 424 \text{ muons hr}^{-1}.$$

When the effects of time dilation are included, the theoretical prediction is in excellent agreement with the experimental result.

From a muon's rest frame, its lifetime is only 2.20×10^{-6} s. How would an observer riding along with the muons, as shown in Fig. 4.8, explain their ability to reach sea level? The observer would measure a severely length-contracted Mt. Washington (in the direction of the relative motion only). The distance traveled by the muons would not be $L_{\text{rest}} = 1907$ m, but rather

$$L_{\text{moving}} = L_{\text{rest}}\sqrt{1 - u^2/c^2} = 1907\ \text{m}\ \sqrt{1 - (0.9952)^2} = 186.6\ \text{m}.$$

Thus it would take $(1.866 \times 10^4\ \text{cm})/(0.9952c) = 6.25 \times 10^{-7}$ s for the muons to travel the length-contracted distance to the detector at sea level, as measured by an observer in the muons' rest frame. That observer would then calculate the number of muons reaching the lower detector to be

$$N = N_0\, e^{-t/\tau} = 563\, e^{-(6.25\times10^{-7}\ \mathrm{s})/(2.20\times10^{-6}\ \mathrm{s})} = 424\ \mathrm{muons\ hr}^{-1},$$

in agreement with the previous result. This shows that an effect due to time dilation as measured in one frame may instead be attributed to length contraction as measured in another frame.

The effects of time dilation and length contraction are both symmetric between two observers in relative motion. Imagine two identical spaceships that move in opposite directions, passing each other at some relativistic speed. Observers onboard each spaceship will measure the other ship's length as being shorter than their own, and the other ship's clocks as running slower. *Both observers are right*, having made correct measurements from their respective frames of reference.

The reader should not think of these effects as being due to some sort of "optical illusion" caused by light taking different amounts of time to reach an observer from different parts of a moving object. The language used in the preceding discussions have involved the *measurement* of an event's spacetime coordinates (x, y, z, t) using meter sticks and clocks located *at that event*, so there is no time delay. Of course, no actual laboratory has an infinite collection of meter sticks and clocks, and the time delays caused by finite light-travel times must be taken into consideration. This will be important in determining the relativistic Doppler shift formula, which follows.

In 1842 the Austrian physicist Christian Doppler showed that as a source of sound moves through a medium (such as air), the wavelength is compressed in the forward direction and expanded in the backward direction. This change in wavelength of any type of wave caused by the motion of the source or the observer is called a **Doppler shift**. Doppler deduced that the difference between the wavelength $\lambda_{\rm obs}$ observed for a moving source of sound and the wavelength $\lambda_{\rm rest}$ measured in the laboratory for a reference source at rest is related to the radial velocity v_r (the component of the velocity directly toward or away from the observer; see Fig. 1.15) of the source through the medium by

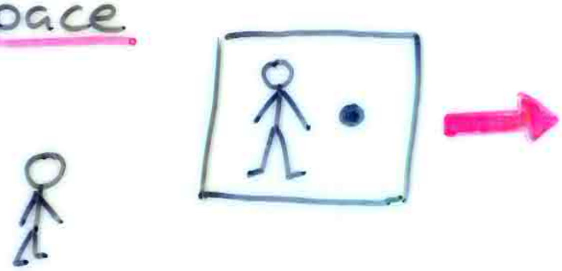
$$\frac{\lambda_{\rm obs} - \lambda_{\rm rest}}{\lambda_{\rm rest}} = \frac{\Delta\lambda}{\lambda_{\rm rest}} = \frac{v_r}{v_s}, \tag{4.30}$$

where v_s is the speed of sound in the medium. However, this expression cannot be precisely correct for light. Experimental results such as those of Michelson and Morley led Einstein to abandon the ether concept, and they demonstrated

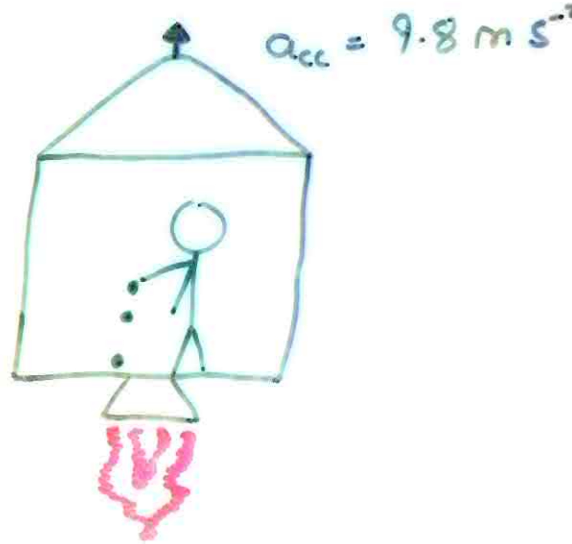
Special Relativity → Uniform Motion

What happens when you are Accelerating?

Free space



Gravity

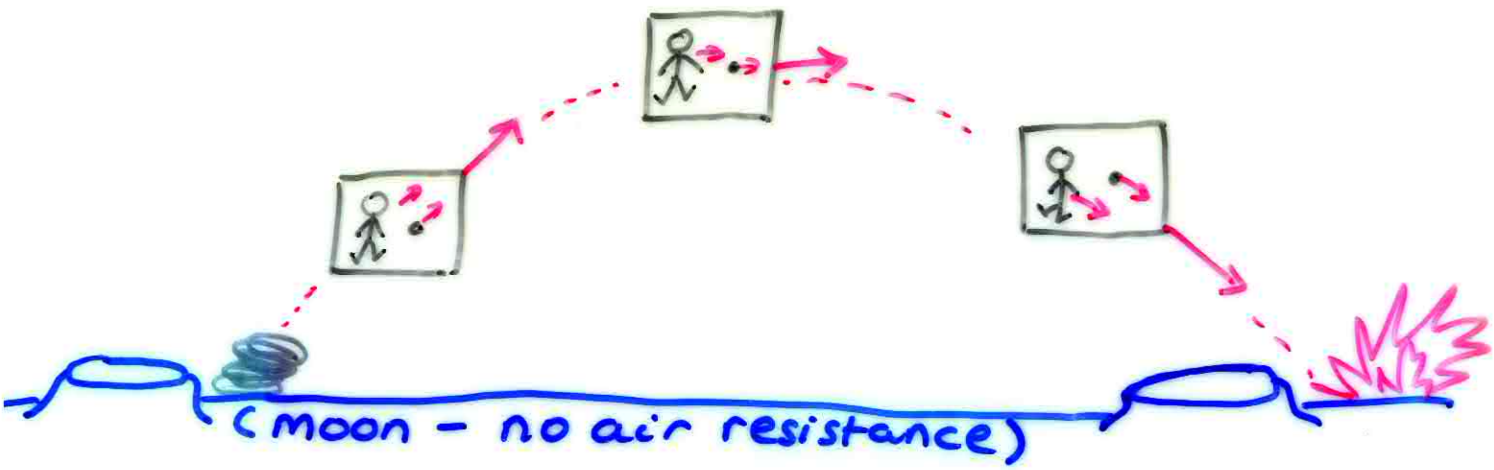


All experiments would give same results

(g-forces)

Weak Equivalence Principle

What about free-fall?

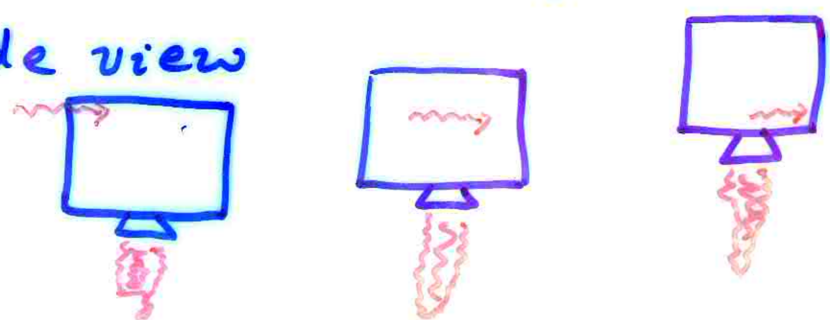


In free-fall, all experiments give the same results as if there is no gravity!

Strong Equivalence Principle

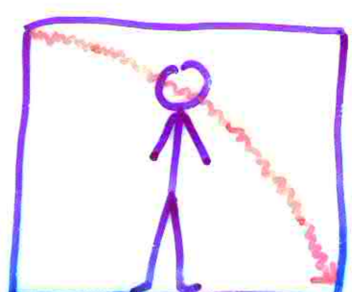
Imagine an accelerating rocket

Outside view



INSIDE view

LIGHT BENDS IN ACCELERATING FRAME



BUT *

SAME MUST BE TRUE IN GRAVITATIONAL FIELD (Eqivalence !!!!)

BUT (AGAIN)

LIGHT TRAVELS THROUGH SPACE-TIME IN STRAIGHT LINES

∴ GRAVITY = CURVED SPACE-TIME

* * General Relativity **

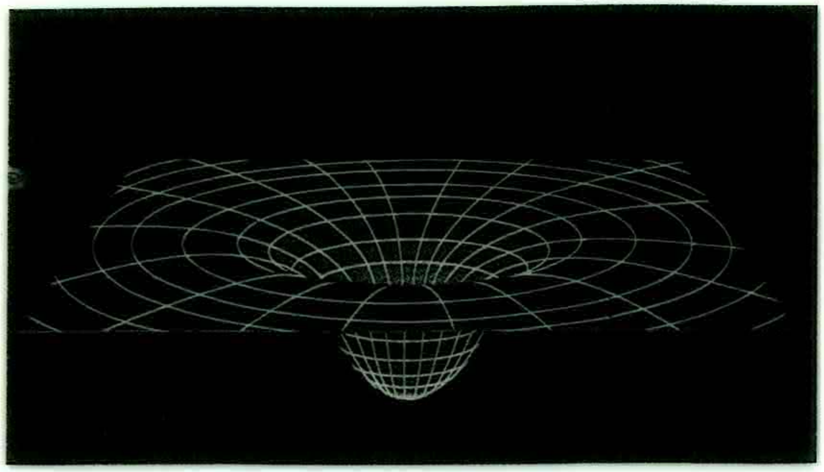
Tests

1. Bending of light
2. Mercury's orbit
3. Gravitational Redshift

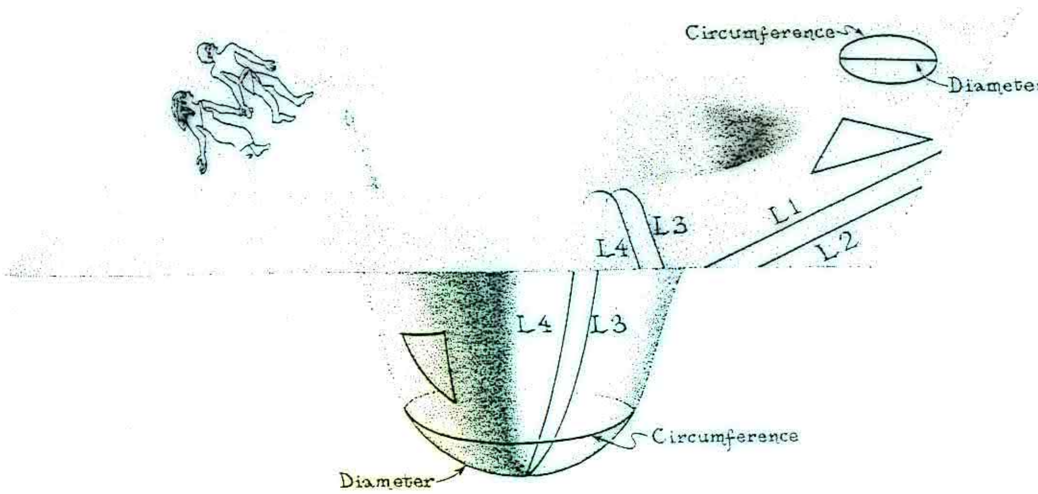
Note G.R. is normally only need for "wierd" objects, such as black holes + neutron stars

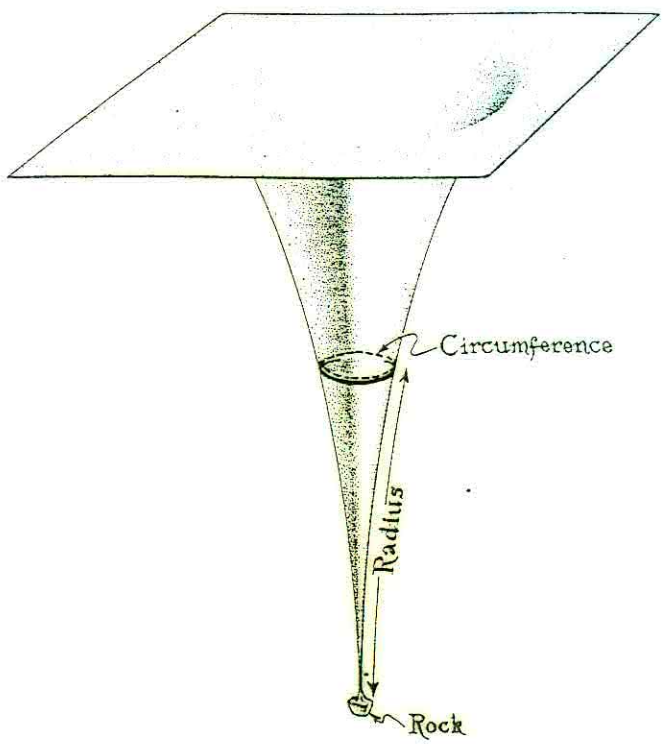
* But *

It is need to describe the Universe as a whole.



3.2 A two-dimensional universe peopled by 2D beings.





Our simple calculation shows that the energy equivalent of even a small mass is very large.

Other relativistic effects include the slowing of moving clocks and the shrinkage of lengths measured in the direction of motion. A detailed discussion of the major consequences of the special theory of relativity is beyond the scope of this book. Instead, we must consider Einstein's second advance, the general theory.

The General Theory of Relativity

In 1916, Einstein published a more general version of the theory of relativity that dealt with accelerated as well as uniform motion. This **general theory of relativity** contained a new description of gravity.

Einstein began by thinking about observers in accelerated motion. Imagine an observer sitting in a spaceship (Figure 5-19). Such an observer cannot distinguish between the force of gravity and the inertial forces produced by the acceleration of the spaceship. This led Einstein to conclude that gravity and motion through space-time are related, a conclusion now known as the equivalence principle:

> **Equivalence principle** Observers cannot distinguish locally between inertial forces due to acceleration and uniform gravitational forces due to the presence of a massive body.

The importance of the general theory of relativity lies in its description of gravity. Einstein concluded that gravity, inertia, and acceleration are all associated with the way space and time are related. This relation is often referred to as curvature, and a one-line description of general relativity explains a gravitational field as a curved region of space-time:

> **Gravity according to general relativity**
> Mass tells space-time how to curve, and the curvature of space-time (gravity) tells mass how to accelerate.

Thus, we feel gravity because the mass of the earth causes a curvature of space-time. The mass of our bodies responds to that curvature by accelerating toward the center of the earth. According to general relativity, all masses cause curvature, and the larger the mass, the more severe the curvature.

Confirmation of the Curvature of Space-Time

Einstein's general theory of relativity has been confirmed by a number of experiments, but two are worth mentioning here because they were among the first tests of the theory. One involves Mercury's orbit, and the other involves eclipses of the sun.

Johannes Kepler understood that the orbit of Mercury is elliptical, but only since 1859 have astronomers known that the long axis of the orbit sweeps around the sun in a motion called precession (Figure 5-20). The total observed precession is 5600.73 seconds of arc per century (as seen from Earth), or about 1.5° per century. This precession is produced by the gravitation of Venus, Earth, and the other planets. However, when astronomers used Newton's description of gravity, they calculated that the precession should amount to only 5557.62 seconds of arc per century. Thus, Mercury's orbit is advancing 43.11 seconds of arc per century faster than Newton's law predicted.

FIGURE 5-19

(a) An observer in a closed spaceship on the surface of a planet feels gravity. (b) In space, with the rockets smoothly firing and accelerating the spaceship, the observer feels inertial forces that are equivalent to gravitational forces.

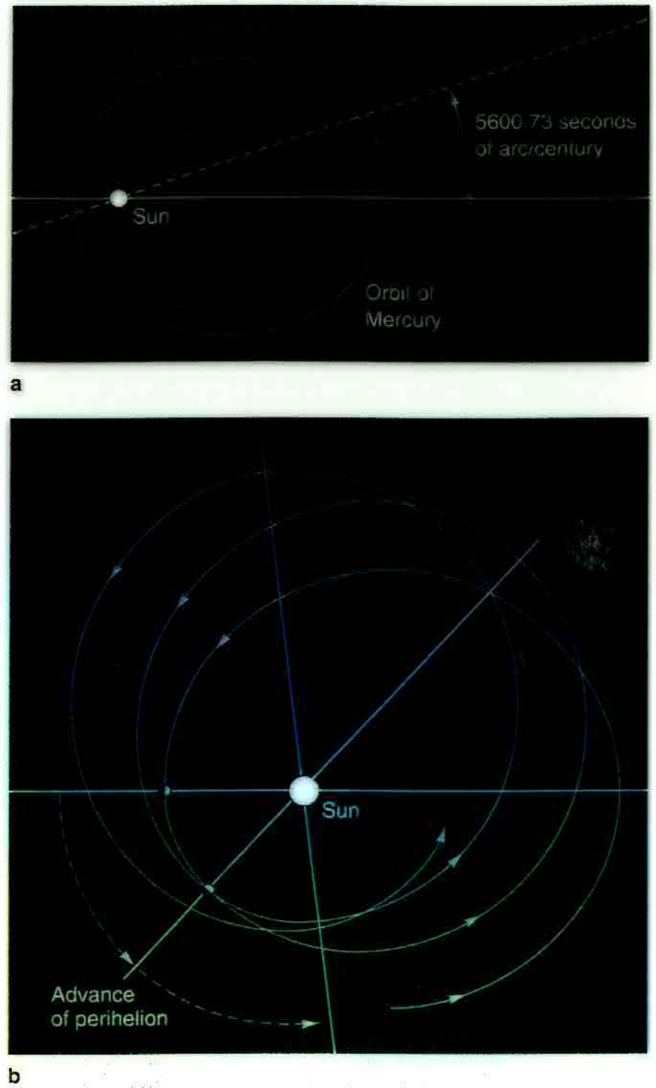


FIGURE 5-20

(a) Mercury's orbit precesses 5600.73 seconds of arc per century—43.11 seconds of arc per century faster than predicted by Newton's laws. (b) Even when we ignore the influences of the other planets, Mercury's orbit is not a perfect ellipse. Curved space-time near the sun distorts the orbit from an ellipse into a rosette. The advance of Mercury's perihelion is exaggerated about a million times in this figure.

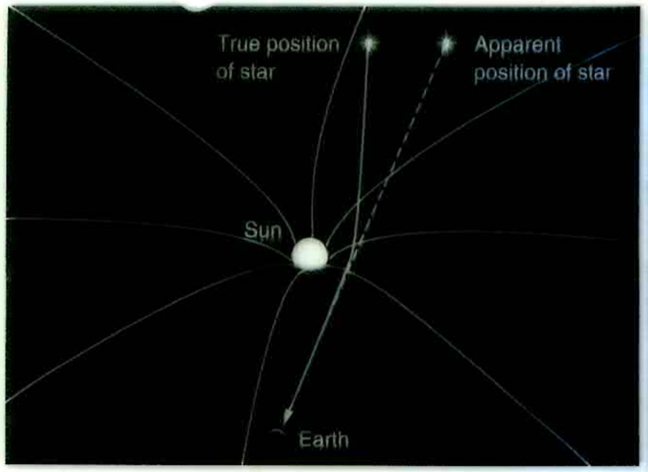


FIGURE 5-21
Like a depression in a putting green, the curved space-time near the sun deflects light from distant stars and makes them appear to lie in slightly different positions.

lations in a putting green (Figure 5-21). Einstein predicted that starlight grazing the sun's surface would be deflected by 1.75 seconds of arc. Starlight passing near the sun is normally lost in the sun's glare, but during a total solar eclipse stars beyond the sun could be seen. As soon as Einstein published his theory, astronomers rushed to observe such stars and thus test the curvature of space-time.

The first solar eclipse following Einstein's announcement in 1916 was June 8, 1918. It was cloudy. The next occurred on May 29, 1919, only months after the end of World War I, and was visible from Africa and South America. British teams went to both Brazil and Príncipe, an island off the coast of Africa. First, they photographed that part of the sky where the sun would be located during the eclipse and measured the positions of the stars on the plates. Then during the eclipse they photographed the same star field with the eclipsed sun located in the middle. After measuring the plates, they found slight changes in the positions of the stars. During the eclipse, the positions of the stars on the plates were shifted outward, away from the sun (Figure 5-22). If a star had been located at the edge of the solar disk, it

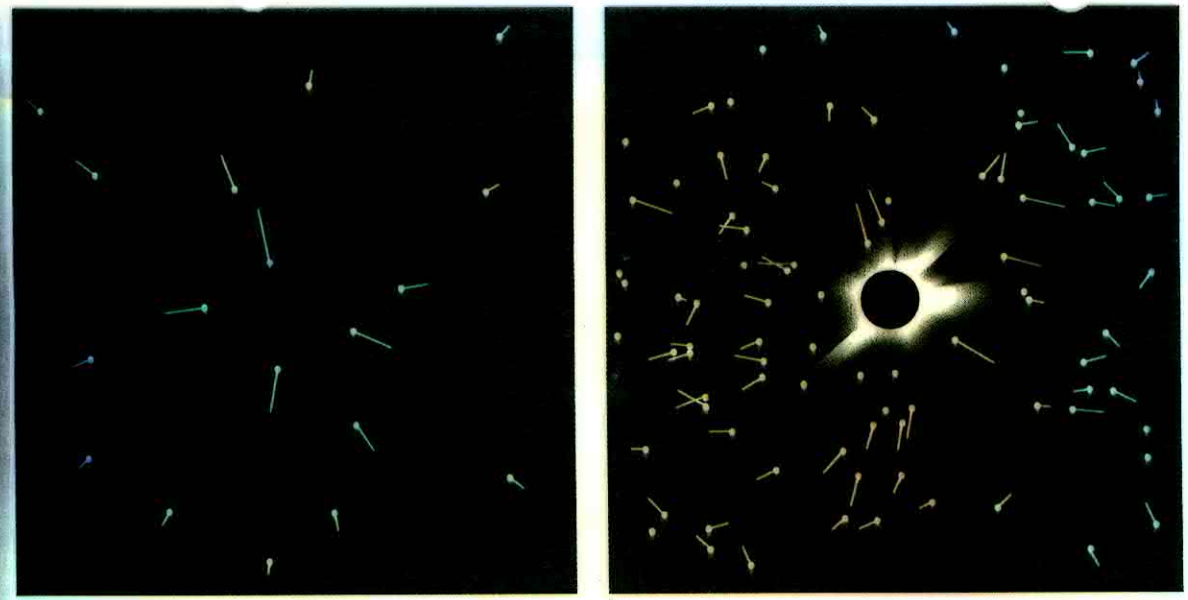


FIGURE 5-22
(a) Schematic drawing of the deflection of starlight by the sun's gravity. Dots show the true positions of the stars as photographed months before. Lines point toward the positions of the stars during the eclipse. (b) Actual data from the eclipse of 1922. Random errors of observation cause some scatter in the data, but in general the stars appear to move away from the sun by 1.77 seconds of arc at the edge of the sun's disk. The deflection of stars is magnified by a factor of 2300 in both (a) and (b).

providing a theory of gravity based on the geometry of curved space-time. Thus, Galileo's inertia and Newton's mutual gravitation are shown to be fundamental properties of space and time.

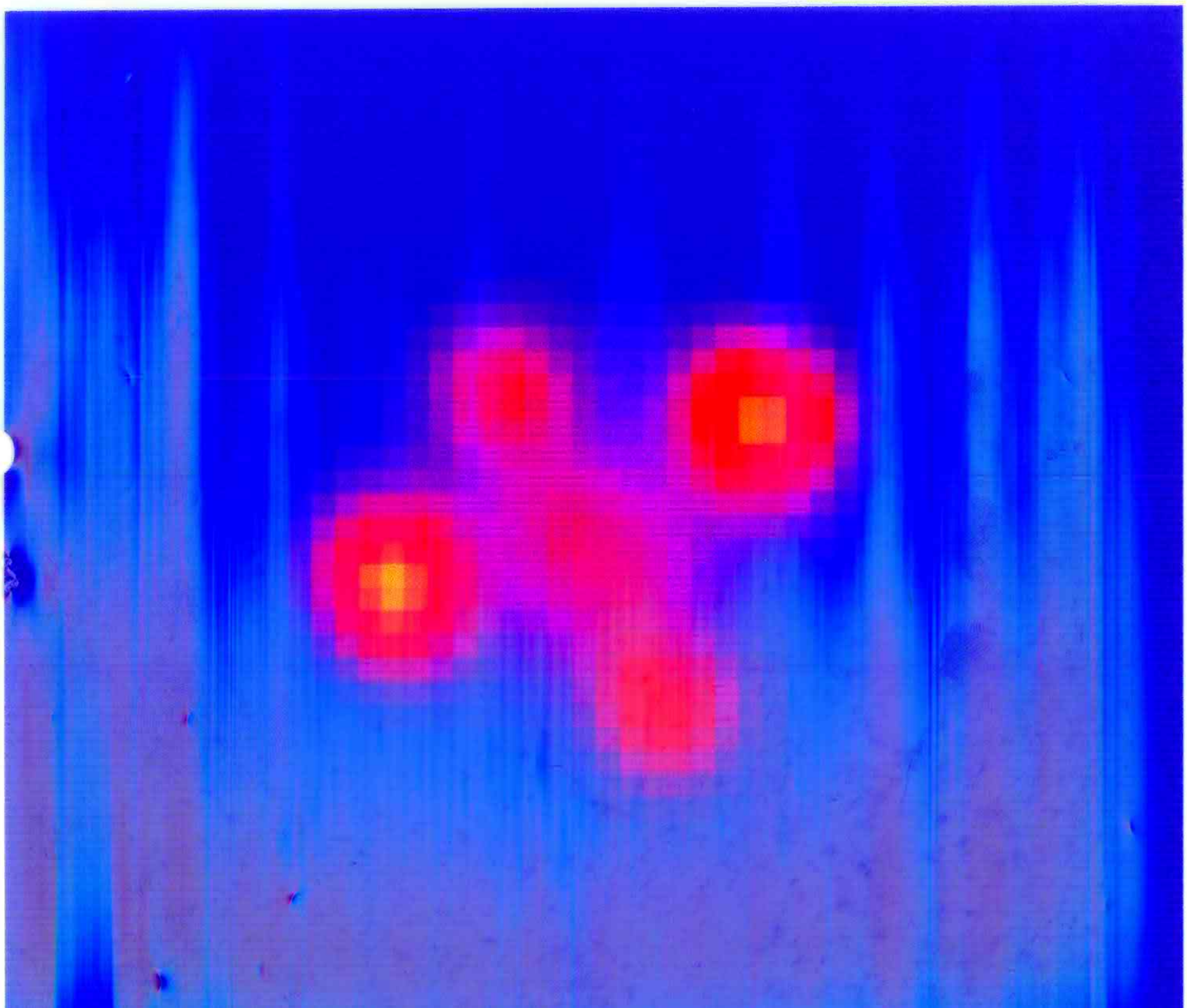
CRITICAL INQUIRY

What does the equivalence principle tell us?

The equivalence principle says that there is no observation we can make inside a closed spaceship to distinguish between uniform acceleration and gravitation. Of course, we could open a window and look outside, but then we would no longer be in a closed spaceship. As long as we make no outside observations, we can't tell

But what about the second postulate of sp
tivity? Why does it have to be true if the first
true? And what does the second postulate tell
the nature of uniform motion? ■

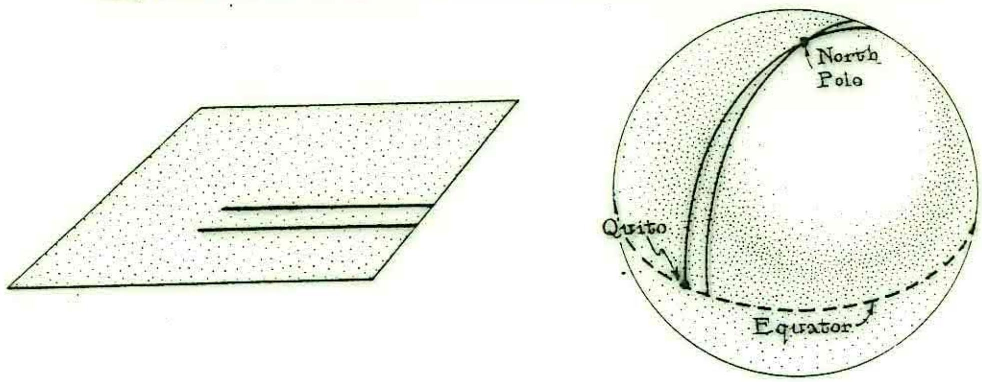
Our discussion of the origin of astronomy beg
the builders of Stonehenge and reaches the m
with Einstein's general theory of relativity. N
we have seen where astronomy came from, we
to see how it helps us understand the nature o
verse. Our first question should be "How do a
get information?" The answer involves the ast
most basic tool, the telescope; that is the subj
next chapter. ■



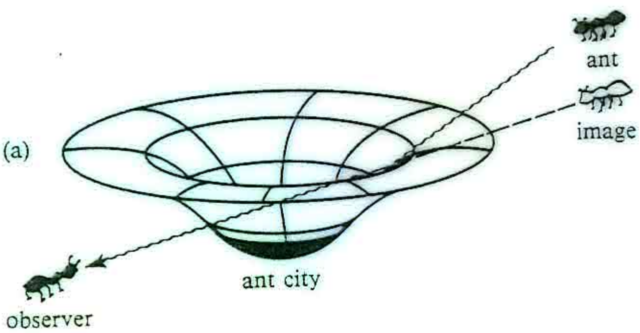

PRC95-14 · ST ScI OPO · April 5, 1995 · W. Couch (UNSW), NASA
Gravitational Lens in Abell 2218
HST · WFPC2

10

2.4 Two straight lines, initially parallel, never cross on a flat surface such as the sheet of paper shown on the left. Two straight lines, initially parallel, will typically cross on a curved surface such as the globe of the world shown on the right.



(a)



(b)